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Inverse Trigonometric Functions

MATHS with easy method by KULDEEP CHAUHAN

Dear students Ch-2 (ITF) is the part of unit-1 and unit-1 is deserves for 8 marks in board exam

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Topic wice Qs. Description References NCERT Text Book XII
(i) Principle value branch table Ex.-2.1 Q.no-11,14
(ii) Properties of Inverse Trigonometric Ex.-2.2 Q.no-7,13,15 Misc. Ex.Q.no-9,10,11,12

Some Important results/concepts

Domain & Range of the Inverse Trigonometric Function :-

	Function	Domain	Range (principal value branch)
(i)	sin^{-1}	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(ii)	cos^{-1}	$[-1,1]$	$[0, \pi]$
(iii)	$cosec^{-1}$	$\mathrm{R}-(-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
(iv)	sec^{-1}	$\mathrm{R}-(-1,1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
(v)	tan^{-1}	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(vi)	cot^{-1}	R	$(0, \pi)$

Properties of Inverse Trigonometric Function

1. (i) $sin^{-1}(\sin \mathrm{x}) = \mathrm{x}$ & $\sin(\sin^{-1}\mathrm{x}) = \mathrm{x}$
   (ii) $cos^{-1}(\cos x) = x$ & $\cos(cos^{-1}x) = x$
   (iii) $tan^{-1}(\tan x) = x$ & $tan(tan^{-1}x) = x$
   (iv) $cot^{-1}(\cot x) = x$ & $\cot(cot^{-1}x) = x$
   (v) $sec^{-1}(\sec x) = x$ & $\sec(sec^{-1}x) = x$
   (v) $cosec\ x^{-1}(cosec\ x) = x$ & $cosec\ (cosec^{-1}) = x$
2. (i) $sin^{-1}x = cosec^{-1}\frac{1}{x}$ & $sin^{-1}x = cosec^{-1}\frac{1}{x}$
   (ii) $cos^{-1}x = sec^{-1}\frac{1}{x}$ & $sec^{-1}x = cos^{-1}\frac{1}{x}$
   (iii) $tan^{-1}x = cot^{-1}\frac{1}{x}$ & $sin^{-1}x = cosec^{-1}\frac{1}{x}$
3. (i) $sin^{-1}x(-x) = -sin^{-1}x$
   (ii) $tan^{-1}(-x) = -tan^{-1}$
   (iii) $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}$
   (iv) $cos^{-1}(-x) = \pi - cos^{-1}$
   (v) $sec^{-1}(-x) = \pi - sec^{-1}$
   (vi) $cot^{-1}(-x) = \pi - cot^{-1}$
4. (i) $sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$
5. (ii) $tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$
   (i) $cosec^{-1}x + sec^{-1}x = \frac{\pi}{2}$
6. $2tan^{-1}x = tan^{-1}\left(\frac{2x}{1-x^2}\right) = cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = sin^{-1}\left(\frac{2x}{1+x^2}\right)$
7. $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)\ if\ xy < 1$

$$tan^{-1}x + tan^{-1}y = \pi + tan^{-1}\left(\frac{x+y}{1-xy}\right)\ if\ xy > 1$$

$$tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right) if\ xy > -1$$

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}$$

$$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

	$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ $\mathrm{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

S.No.	Function	+ive quadrant –ive	
1	$\sin^{-1}x$	I	IV(0-θ)
2	$\cos^{-1}x$	I	II(π-θ)
3	$\tan^{-1}x$	I	IV(0-θ)
4	$\cot^{-1}x$	I	II(π-θ)
5	$\sec^{-1}x$	I	II(π-θ)
6	$\mathrm{cosec}^{-1}x$	I	IV (0-θ)

TRIGNOMETRIC FORMULA

Addition and subtraction of angle

$(i)\quad \sin(x+y) = sinxcosy + cosxsiny$

$(ii)\quad \sin(x-y) = sinxcosy - cosxsiny$

$(iii)\quad \cos(x+y) = cosxcosy - sinxsiny$

$(iv)\quad \cos(x-y) = cosxcosy + sinxsiny$

$$(v)\quad \tan(x+y) = \frac{tanx + tany}{1 - tanx.tany}$$

$$(vi)\quad \tan(x-y) = \frac{tanx - tany}{1 + tanx.tany}$$

$$(vii)\quad \cot(x+y) = \frac{cot\,x.coty - 1}{cotx + coty}$$

$$(viii)\quad \cot(x-y) = \frac{cotx.\cot y + 1}{\cot y - \cot x}$$

Addition and subtraction of function

$$(i)\quad sinx + siny = 2sin\frac{(x+y)}{2}cos\frac{(x-y)}{2}$$

(ii) $sinx - siny = 2cos\frac{(x+y)}{2}sin\frac{(x-y)}{2}$

(iii) $cosx + cosy = 2cos\frac{(x+y)}{2}cos\frac{(x-y)}{2}$

(iv) $cosx + cosy = -2sin\frac{(x+y)}{2}sin\frac{(x-y)}{2}$

Coefficient of angle

(i) $sin2x = 2sinx.cosx = \frac{2tanx}{1+tan^2x}$

(ii) $cos2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x = \frac{1-tan^2x}{1+tan^2x}$

(iii) $tan2x = \frac{2tanx}{1-tan^2x}$

(iv) $sin3x = 3sinx - 4sin^3x$

(v) $cos3x = 4cos^3x - 3cosx$

(vi) $tan3x = \frac{3tanx - tan^3x}{1-3tan^2x}$

Multiplication of function

(i) $2sinx.cosy = \sin(x+y) + \sin(x-y)$

(ii) $2cosx.cosy = \cos(x+y) + \cos(x-y)$

(iii) $2\sin x.siny = \cos(x-y) - \cos(x+y)$

Exercise-A

1. *Find the value of* $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.
2. *Evaluate that:* $-\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$
3. *Prove that:* $-\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=7$
4. *Find the value of* $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.
5. *Find the value of* $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.
6. *Show that* $2\tan^{-1}(-3)=-\frac{\pi}{2}+\tan^{-1}\left(-\frac{4}{3}\right)$.
7. *Find the real solution of* $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$.
8. *Find the value of* $\sin\left(2\tan^{-1}\frac{1}{3}\right)+\cos\left(\tan^{-1}2\sqrt{2}\right)$.
9. *If* $2\tan^{-1}(\cos\theta)=\tan^{-1}(2\,cosec\,\theta)$, *then show that* $\theta=\frac{\pi}{4}$
10. *Show that* $\cos\left(2\tan^{-1}\frac{1}{7}\right)=\sin\left(4\tan^{-1}\frac{1}{3}\right)$
11. *Solve the equation* $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$

LONG ANSWER TYPE QUESTION

1. *Prove that* $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$
2. *Find the simplified form of* $\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$, *where,* $x\in\left[-\frac{3\pi}{4},\frac{\pi}{4}\right]$.
3. *Prove that* $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{77}{85}$.
4. *Show that* $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$.
5. *Show that* $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\sin^{-1}\frac{1}{\sqrt{5}}$.
6. *Find the value of* $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$.

8. *Show that* $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$
9. *If* $a_1, a_2, a_3 \ldots\ldots\ldots.. a_n$ *is an arithmetic progression with common difference d, then evaluate the following expression.*

$$tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\cdots+tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

9. $sin\left[\frac{\pi}{3}-sin^{-1}\left(-\frac{1}{2}\right)\right]$
10. *If* $tan^{-1}x+tan^{-1}y=\frac{\pi}{4}$, *find the value of* $x+y+xy$
11. *Find the value of* $tan^{-1}x+tan^{-1}\frac{1}{x}, x<0$
12. *Show that,* $sec^2(tan^{-1}2)cosec^2(cot^{-1}3)=15$
13. *Show that,* $sec^2(tan^{-1}3)+cosec^2(cot^{-1}4)=27$
14. *Show that,* $tan^{-1}2+tan^{-1}3=\frac{3\pi}{4}$
15. *Find the value of* $tan^{-1}\left[2cos\left\{2\,sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$
16. *Find the value of* $tan\left[\frac{1}{2}cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$
17. *Show that,* $tan\left[2\,tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]$
18. *Solve for* x: $tan^{-1}x+tan^{-1}(1-x)=cot^{-1}\frac{7}{9}$
19. *If* $tan^{-1}x+tan^{-1}y+tan^{-1}z=\pi$, *then prove that* $x+y+z=xyz$
20. *If* $cot^{-1}x+cot^{-1}y+cot^{-1}z=\pi$, *then prove that* $xy+yz+zx=1$
21. *Show that* : $sin^{-1}\frac{4}{5}+2\,tan^{-1}\frac{1}{3}=\frac{\pi}{2}$
22. *Show that* : $2\,tan^{-1}\left(\frac{1}{3}\right)+tan^{-1}\left(\frac{1}{7}\right)=\frac{\pi}{4}$
23. $tan^{-1}\left(\frac{cos\,x-sin\,x}{cos\,x+sin\,x}\right)$
24. $tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$
25. $tan^{-1}\left(\frac{a\,cos\,x-b\,sin\,x}{b\,cos\,x+a\,sin\,x}\right)$
26. $tan^{-1}\left[\frac{3a^2x-x^3}{a(a^2-3x^2)}\right]$
27. If $\alpha=\sin^{-1}\left(\frac{7}{25}\right)$ *and* $\beta=\cos^{-1}\left(\frac{3}{5}\right)$, *find the value of* $\sin(\alpha+\beta)$

28. $If\ \cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\ \theta, then\ prove\ that$

$9x^2-12xy\cos\theta+4y^2=36(\sin\theta)^2$

29. $\cos^{-1}x-\sin^{-1}x=0\ then\ find\ value\ of\ x.$

30. $Write\ the\ range\ of\ the\ principal\ branch\ of\ \sec^{-1}x\ defined\ on\ the\ domain\ R(-1,1)$

31. $Solve\ for\ x:\ \cos(2\sin^{-1}x)=\frac{1}{9}$

32. $Solve\ for\ x:\ \tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

33. $Prove\ That:\ \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

34. $If\ \cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi. Prove\ that\ x^2+y^2+z^2\ 2xyz=\ 1$

35. $Prove\ that:\ \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

36. $Prove\ that:\ \tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

37. $Prove\ that:\ \tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

37. $If\ \tan^{-1}2\ and\ \tan^{-1}3\ be\ two\ angle\ of\ triangle, then\ find\ the\ third\ angle\ of\ triangle$

38. $Solve\ for\ x:\ \sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

39. $Solve:-\ \sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\frac{\pi}{2}$

40. $Solve\ for\ x:2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), 0<\frac{\pi}{2}$

41. $x:\ \tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right)$

42. $Solve\ for\ x:\ \tan^{-1}\left(\frac{1-x}{1+x}\right)-\frac{1}{2}\tan^{-1}x=0$

43. $Solve\ for\ x:\ \cos^{-1}x+\sin^{-1}\left(\frac{x}{2}\right)=\frac{\pi}{2}$

44. $Prove\ that,\quad \cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$

45. $Solve\ for\ x:\ \tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\tan^{-1}(-7)$

46. $Find\ the\ positive\ integral\ solutions\ of\ the\ equation,$

$$\tan^{-1}x+\cos^{-1}\frac{y}{\sqrt{1+y^2}}=\sin^{-1}\frac{3}{\sqrt{10}}$$

47. $Prove\ that:\ tan^{-1}\sqrt{\frac{xr}{yz}} + tan^{-1}\sqrt{\frac{yr}{zx}} + tan^{-1}\sqrt{\frac{zr}{xy}} = \pi\ where,\ r = x + y + z$